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Peeking into the AI Language Modeling Black Box

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European Union's Horizon 2020 research and innovation programme
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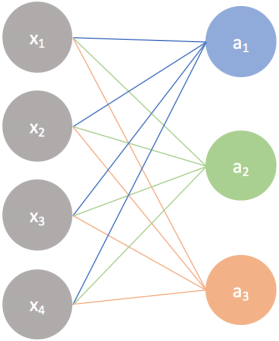
Outline

Outline

Neural Networks

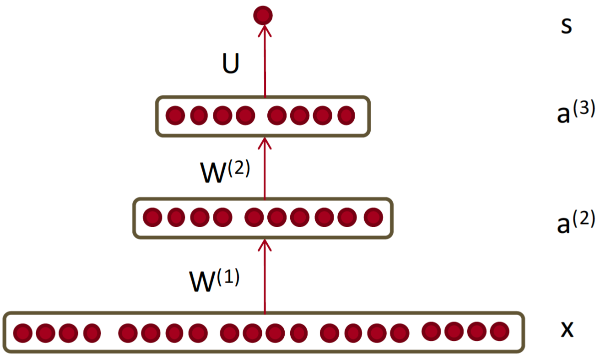


$$\begin{bmatrix} w_1 & w_2 & w_3 & w_4 \\ w_1 & w_2 & w_3 & w_4 \\ w_1 & w_2 & w_3 & w_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} b \\ b \\ b \end{bmatrix} = \begin{bmatrix} w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b \\ w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b \\ w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b \end{bmatrix} \xrightarrow[\text{activation}]{} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Credit: Jeremy Jordan

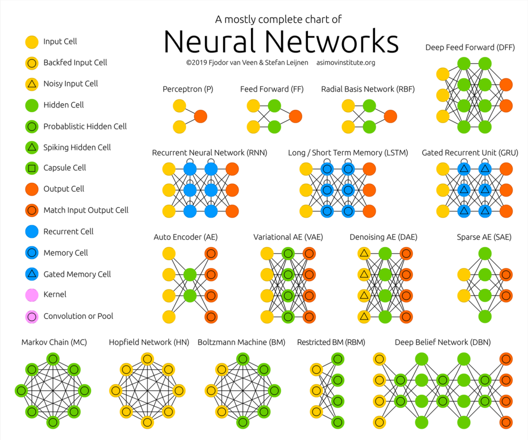
Deep Neural Nets (DNNs)

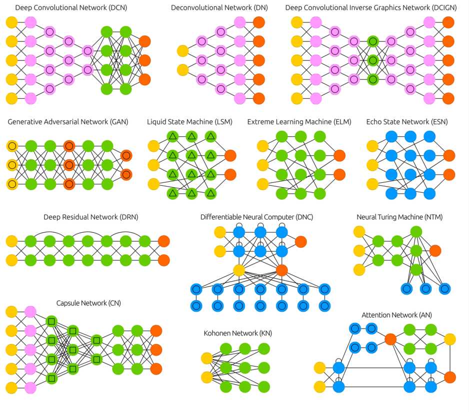
$$
\begin{aligned}
x &= z^{(1)} = a^{(1)} \\
z^{(2)} &= W^{(1)} x + b^{(1)} \\
a^{(2)} &= f\left(z^{(2)}\right) \\
z^{(3)} &= W^{(2)} a^{(2)} + b^{(2)} \\
a^{(3)} &= f\left(z^{(3)}\right) \\
s &= U^T a^{(3)}
\end{aligned}
$$



Credit: Manning & Socher, Stanford CS224n course, 2017

The Family of DNNs





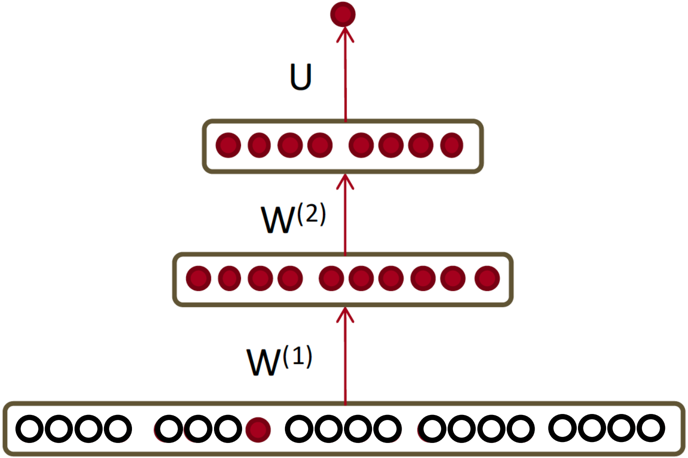
Credit: https://www.asimovinstitute.org/neural-network-zoo/

DNNs and Natural Language I

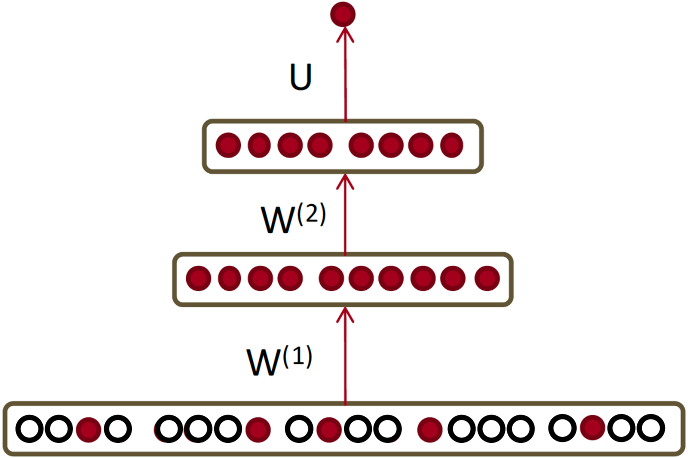
Index	Word
...	...
535	nearly
536	shares
537	member
538	campaign
539	media
540	needs
541	why
542	house
543	issues
544	costs
545	fire
...	...

$$v_{house} = (\underbrace{0, 0, 0, 0, 0, 0, 0, 0, \dots, 0, \overbrace{1}^{542^{\text{nd}}\ \text{position}}, 0, \dots, 0, 0, 0, 0, 0, 0, 0, 0}_{3\ \text{million dimensions}})$$

DNNs and Natural Language II



DNNs and Natural Language II



Outline

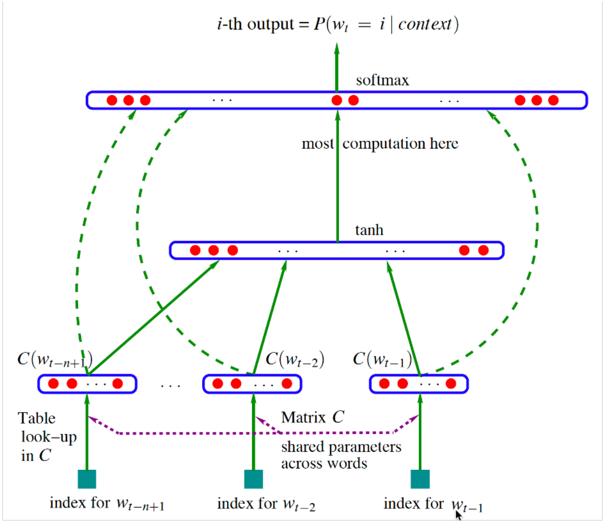
DNNs and Language

Word Embeddings

How Does It Work?

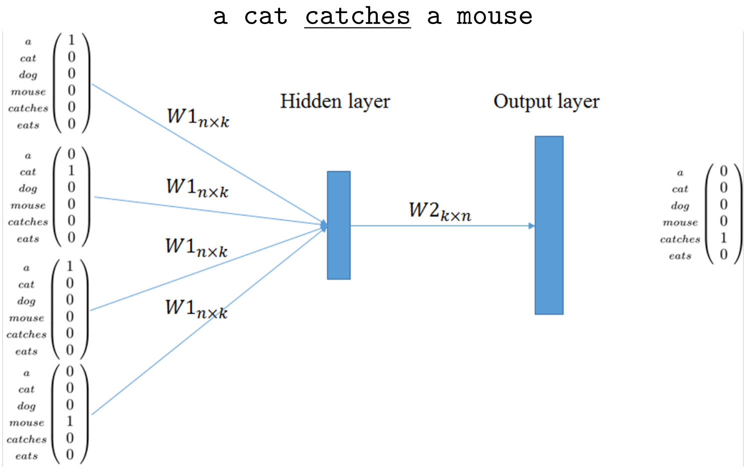
Discussion

Early Neural Language Models



(Bengio et al., 2003)

Word Embeddings: word2vec



Credit: Ferrone et al., 2017

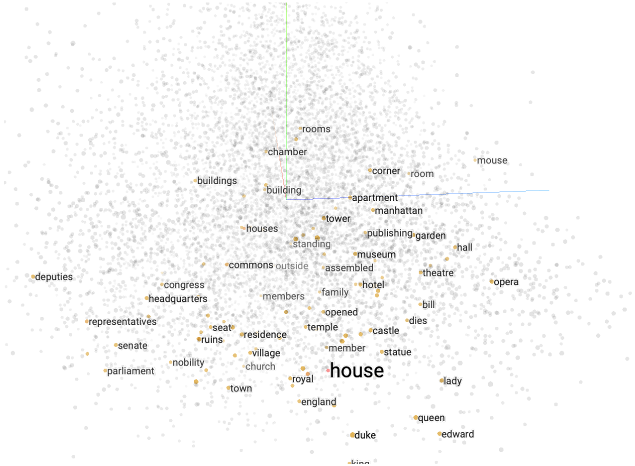
Dense Vector Representations

$$v_{house} = (\underbrace{0,0,0,0,0,0,0,0,\ldots,0,\overbrace{1}^{542^{\text{nd}}\text{ position}},0,\ldots,0,0,0,0,0,0,0,0}_{\text{million dimensions}})$$

$$v_{house} = (\underbrace{0.157227, -0.0708008, 0.0539551, \ldots, -0.041748, 0.00982666, -0.00494385, -0.032959}_{300\text{ dimensions}})$$

Word Embeddings: Similarity

house	cosine distance
houses	0.292761
bungalow	0.312144
apartment	0.3371
bedroom	0.350306
townhouse	0.361592
residence	0.380158
mansion	0.394181
farmhouse	0.414243
duplex	0.424206
homes	0.43802



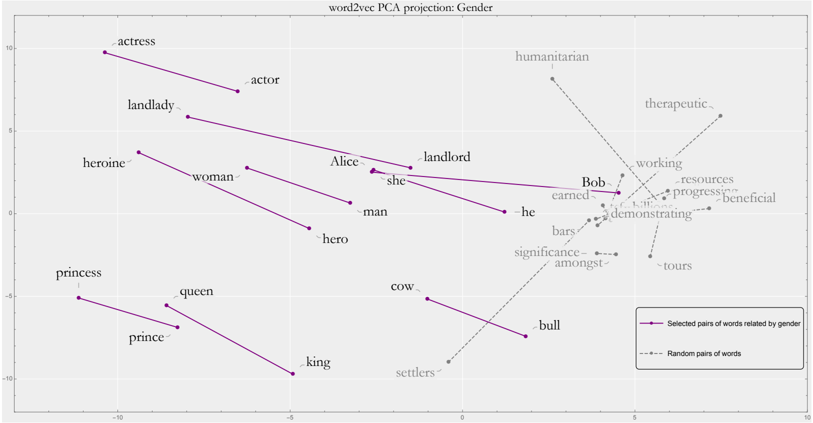
(https://projector.tensorflow.org)

Word Embeddings: Analogy

$$v_{house} - v_{city} + v_{countryside} \approx v_{farmhouse}$$

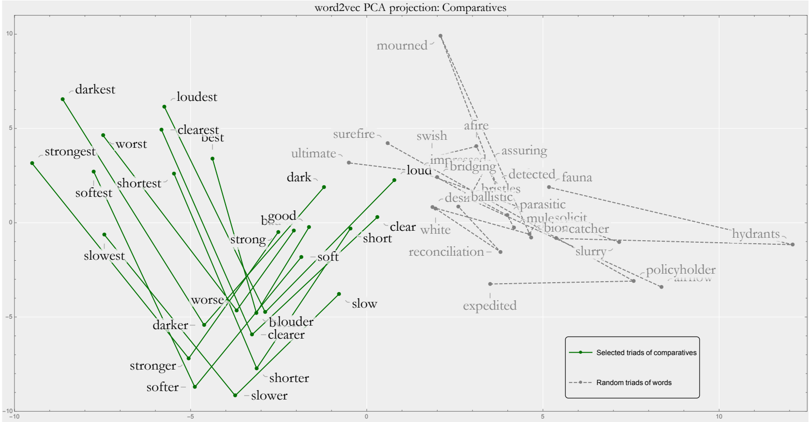
Word Embeddings: Analogy

$$v_{king} - v_{queen} \approx v_{hero} - v_{heroine}$$

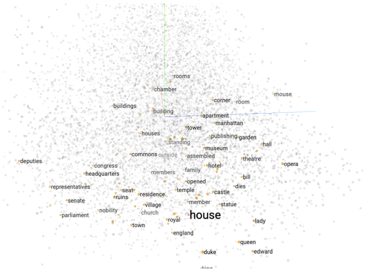


Word Embeddings: Analogy

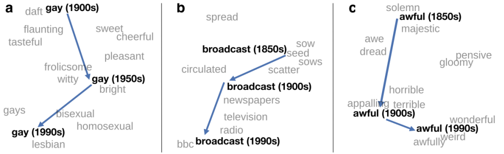
$$v_{good} - v_{better} \approx v_{soft} - v_{softer}$$



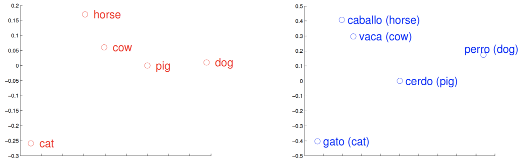
Embedding Space



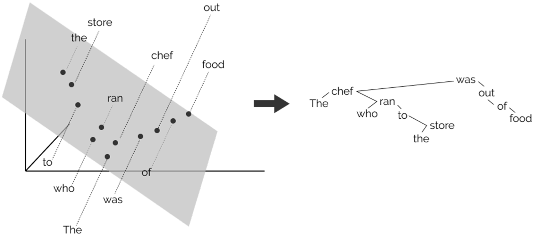
(https://projector.tensorflow.org)



(Hamilton et al., 2016)



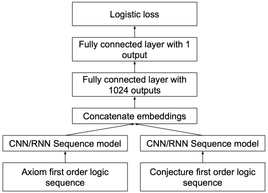
(Mikolov, Sutskever, et al., 2013)



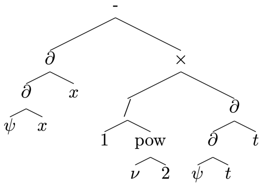
(https://nlp.stanford.edu/~johnhew/structural-probe.html)

Neural LMs Applications to Mathematics

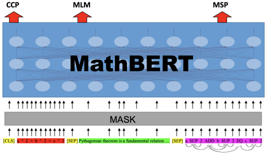
- Proof-Oriented
  - Bansal et al., 2019; Kaliszyk et al., 2017.
- Object-Oriented
  - Blechschmidt and Ernst, 2021; Charton, 2021; d'Ascoli et al., 2022; Lample and Charton, 2019; Li et al., 2021; Ryskina and Knight, 2021
- Skill-Oriented
  - Brown et al., 2020; Peng et al., 2021; Shen et al., 2021
- Heuristic-Oriented
  - Davies et al., 2021



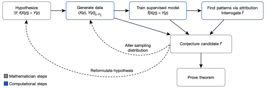
(Alemi et al., 2016)



(Lample and Charton, 2019)

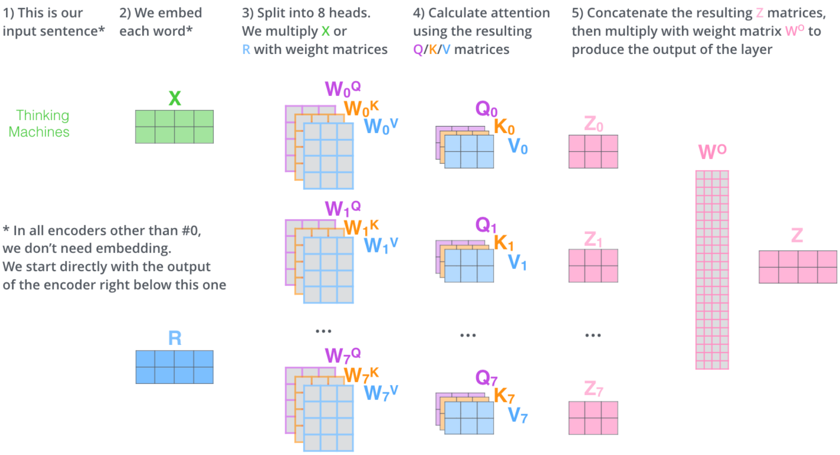


(Peng et al., 2021)



(Davies et al., 2021)

Attention Mechanism: Transformers



Credit: J. Alammar, *The Illustrated Transformer*

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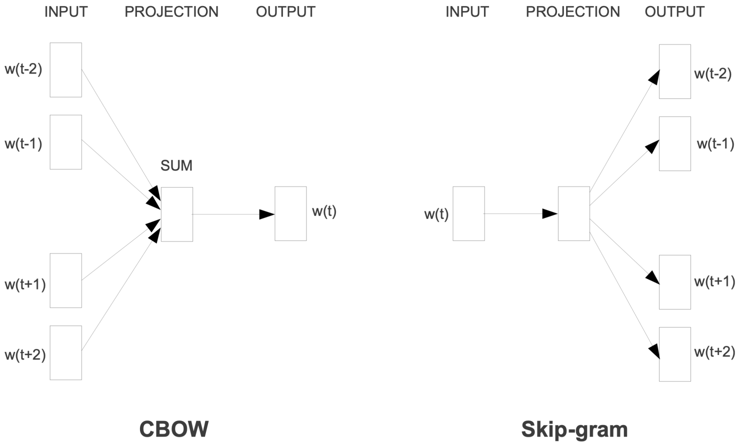
word2vec Models



(Mikolov, Chen, et al., 2013)

word2vec as Implicit Matrix Factorization

(Levy and Goldberg, 2014)

$$\log \sigma(\vec{w} \cdot \vec{c}) + k \cdot \mathbb{E}_{c_N \sim P_D}[\log \sigma(-\vec{w} \cdot \vec{c}_N)]$$

Where:

$\vec{w}$ = vector representation for word w

$\vec{c}$ = vector representation for context c

$\sigma(x)$ = $\frac{1}{1+e^{-x}}$

k = number of “negative” (arbitrary) samples

c_N = arbitrary context sampled from P_D

$P_D(c)$ = empirical unigram distribution of c in the data D, i.e. $\frac{\#(c)}{|D|}$

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word2vec as Implicit Matrix Factorization

(Levy and Goldberg, 2014)

$$\ell = \sum_{w \in V_w} \sum_{c \in V_c} \#(w, c)\left(\log \sigma(\vec{w} \cdot \vec{c}) + k \cdot \mathbb{E}_{c_N \sim P_D}[\log \sigma(-\vec{w} \cdot \vec{c}_N)]\right)$$

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$$\frac{\partial \ell}{\partial (\vec{w} \cdot \vec{c})} = 0$$

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$$\frac{\partial \ell}{\partial(\vec{w} \cdot \vec{c})} = 0 \quad \text{when} \quad \vec{w} \cdot \vec{c} = \log\left(\frac{\#(w,c) \cdot |D|}{\#(w) \cdot \#(c)}\right) - \log k$$

word2vec as Implicit Matrix Factorization

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$$= PMI(w,c) - \log k$$

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Additional constraint: $\vec{w}$ and $\vec{c}$ should be low dimensional

word2vec as Implicit Matrix Factorization

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$$= PMI(w,c) - \log k$$

Additional constraint: $\vec{w}$ and $\vec{c}$ should be low dimensional

There exists an exact solution …

Singular Value Decomposition (SVD)

$$M = U\Sigma V^*$$

Where:

$$\begin{aligned} M &= m \times n \text{ (real or complex) matrix} \\ U &= m \times m \text{ unitary matrix} \\ \Sigma &= m \times n \text{ non-negative real rectangular diagonal matrix} \\ V^* &= \text{conjugate transpose of } V\text{, a } n \times n \text{ unitary matrix} \end{aligned}$$

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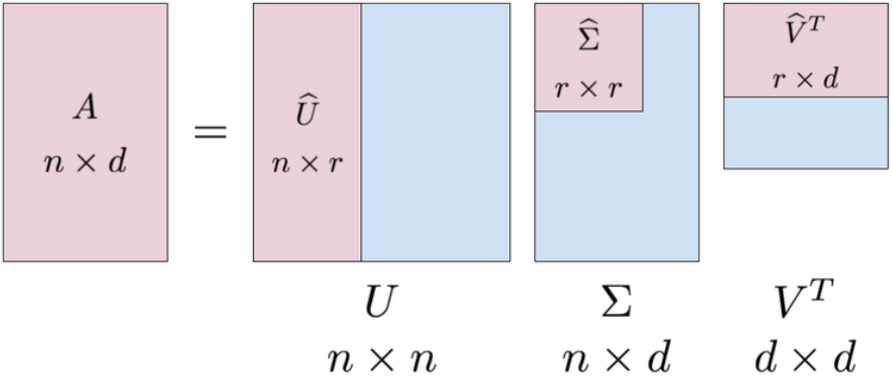
In particular:

- The columns of U (left singular vectors) are eigenvectors of MM^*
- The rows of V^* (right singular values) are eigenvectors of M^*M
- The non-zero elements of Σ (non-zero singular values) are the square roots of the non-zero eigenvalues of MM^* or M^*M

Singular Value Decomposition (SVD)

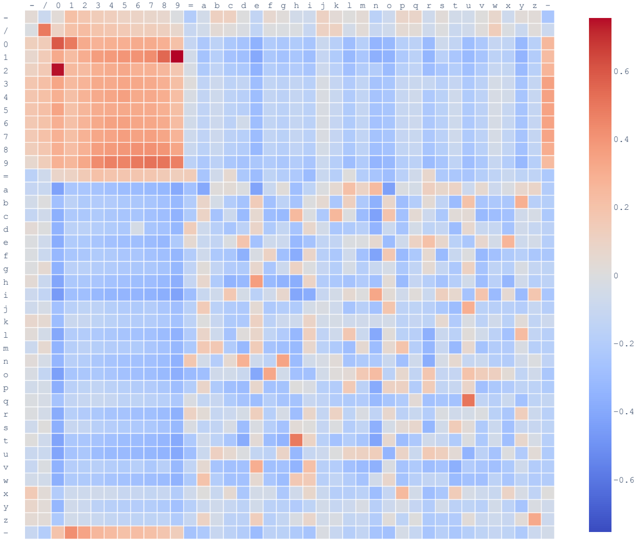
$$M = U\Sigma V^*$$



Credit: Angela Ju

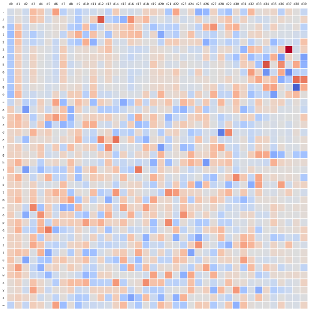
Example: Characters in Wikipedia

$$PMI(w,c) = \log \frac{p(w,c)}{p(w)p(c)}$$

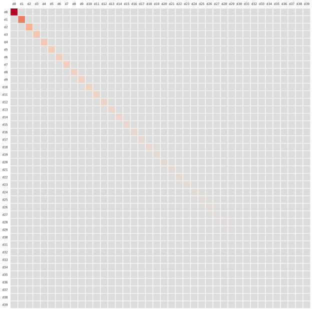


Example: Characters in Wikipedia

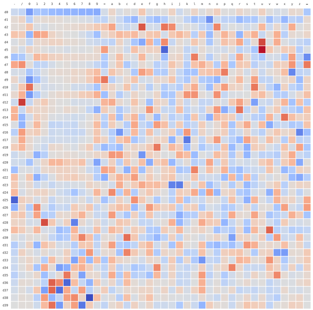
U



Σ

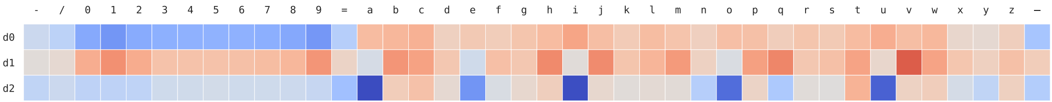


V^*

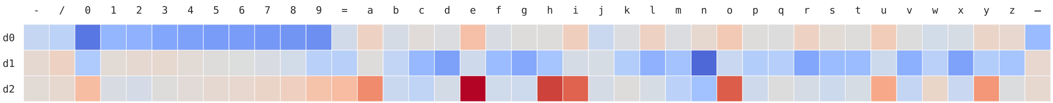


Example: Characters in Wikipedia

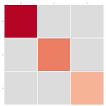
Left Singular Vectors:



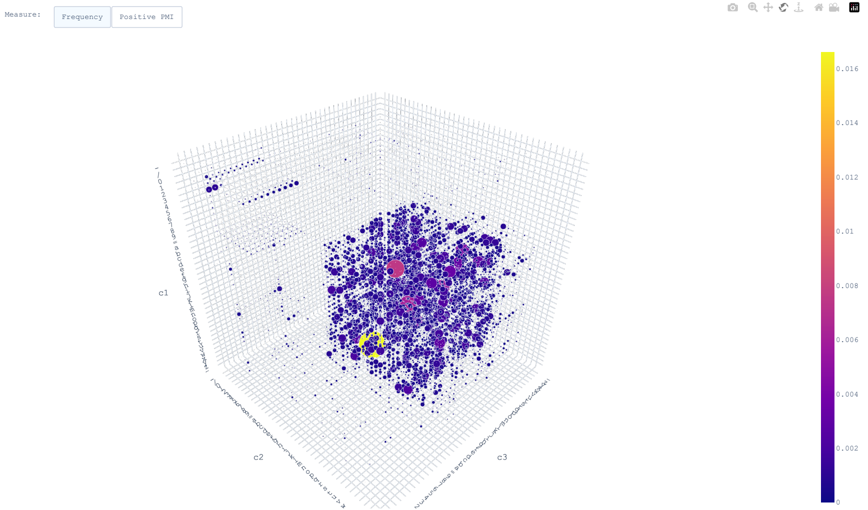
Right Singular Vectors:



Singular Values:



Generalization



Outline

Discussion

- Elementary properties of neural NLP do not depend on the neural nature of models
- There seem to be algebraic structures underlying NL data
- Explicit and symbolic representations could be built upon such structures
- Such representations could be used to explore the capabilities and limits of current neural models
- They could also provide new interpretability principles and techniques
- We need new tools at the interface between algebra and statistics

Reference Paper

- J. L. Gastaldi. Why Can Computers Understand Natural Language? In: *Philosophy & Technology* 34.1 (2021), pp. 149–214.

Related Papers

- J. L. Gastaldi and L. Pellissier. The calculus of language: explicit representation of emergent linguistic structure through type-theoretical paradigms In: *Interdisciplinary Science Reviews* 46.4 (2021), pp. 569–590.
- J. L. Gastaldi, Content from Expressions: The Place of Textuality in Deep Learning Approaches to Mathematics Under review at *Synthese. SI: Linguistically Informed Philosophy of Mathematics.* Fisseni, B., Kant, D., Sarikaya, D. and Schröder, B. (Eds.).

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